J. DIFFERENTIAL GEOMETRY
4 (1970) 237-244

SUBMANIFOLDS OF COSYMPLECTIC MANIFOLDS

G. D. LUDDEN

1. Introduction

Recently B. Smyth [6] has classified those complex Einstein hypersurfaces of a Kaehler manifold of constant holomorphic curvature. This paper was followed by the papers of Chern [2], Nomizu and Smyth [4], Kobayashi [3] and others researching this problem. Yano and Ishihara [7] have studied the analogous problem for Sasakian manifolds, i.e., they have studied invariant Einstein (or η-Einstein) submanifolds of codimension 2 of a normal contact manifold of constant curvature. The result of Smyth rests on the fact that the hypersurface is locally symmetric. We show in this paper that a normal contact manifold which is η-Einsteinian but not Einsteinian cannot be locally symmetric. Thus, since an invariant submanifold of codimension 2 in a normal contact manifold is itself a normal contact manifold, the η-Einstein case studied by Yano and Ishihara will not yield to a study similar to that of Smyth.

Let $\tilde{M}$ be a normal contact manifold or a cosymplectic manifold of constant $\tilde{\phi}$-sectional curvature, and M an invariant submanifold of codimension 2. The main purpose of this paper is to study the case where M is η-Einsteinian. In particular, we show that if $\tilde{M}$ is cosymplectic then M is locally symmetric. This suggests that a classification similar to that of Smyth may be obtained in this case.

2. Almost contact manifolds

Let $\tilde{M}$ be a C^∞-manifold and $\tilde{\phi}$ a tensor field of type (1, 1) on $\tilde{M}$ such that

$$\tilde{\phi}^2 = -I + \tilde{\xi} \otimes \tilde{\eta} ,$$

where I is the identity transformation, $\tilde{\xi}$ a vector field, and $\tilde{\eta}$ a 1-form on $\tilde{M}$ satisfying $\tilde{\phi}\tilde{\xi} = \tilde{\eta} \circ \tilde{\phi} = 0$ and $\tilde{\eta}(\tilde{\xi}) = 1$. Then $\tilde{M}$ is said to have an *almost contact structure*. It is known that there is a positive definite Riemannian metric $\tilde{g}$ on $\tilde{M}$ such that $\tilde{g}(\tilde{\phi}X, Y) = -\tilde{g}(X, \tilde{\phi}Y)$ and $\tilde{g}(\tilde{\xi}, \tilde{\xi}) = 1$, where X and Y are vector fields on $\tilde{M}$. Define the tensor $\tilde{\Phi}$ by $\tilde{\Phi}(X, Y) = \tilde{g}(X, \tilde{\phi}Y)$. Then $\tilde{\Phi}$ is a 2-form. If $[\tilde{\phi}, \tilde{\phi}] + d\tilde{\eta} \otimes \tilde{\xi} = 0$, where $[\tilde{\phi}, \tilde{\phi}](X, Y) = \tilde{\phi}^2[X, Y] + [\tilde{\phi}X, \tilde{\phi}Y] - \tilde{\phi}[\tilde{\phi}X, Y] - \tilde{\phi}[X, \tilde{\phi}Y]$, then the almost contact structure is said to be *normal*. If $\tilde{\Phi} = d\tilde{\eta}$, the almost contact structure is a *contact structure*.

Communicated by K. Yano, June 30, 1969.

A normal almost contact structure such that $\tilde{\Phi}$ is closed and $d\tilde{\eta} = 0$ is called *cosymplectic structure*. It can be shown [1] that the cosymplectic structure is characterized by

$$\tilde{\nabla}_X \tilde{\phi} = 0 \quad \text{and} \quad \tilde{\nabla}_X \tilde{\eta} = 0 \, , \tag{2.1}$$

where $\tilde{\nabla}$ is the connection of $\tilde{g}$. Henceforth, we assume $\tilde{M}$ possesses a normal contact (Sasakian) structure or a cosymplectic structure. We note here that in a Sasakian manifold

$$(\tilde{\nabla}_X \tilde{\phi})Y = \tilde{\eta}(Y)X - \tilde{g}(X, Y)\tilde{\xi} \, . \tag{2.2}$$

The *curvature operator* $\tilde{R}$ of $\tilde{g}$ is defined by $\tilde{R}_{XY}Z = [\tilde{\nabla}_X, \tilde{\nabla}_Y]Z - \tilde{\nabla}_{[X,Y]}Z$ and the *Ricci tensor* $\tilde{S}$ is the trace of the mapping $X \to \tilde{R}_{XY}W$. If X and Y form an orthonormal basis of a 2-plane of $\tilde{M}$, the *sectional curvature* $\tilde{K}(X, Y)$ of this plane is given by $\tilde{g}(\tilde{R}_{XY}X, Y)$. If X is a unit vector which is orthogonal to $\tilde{\xi}$, we say that X and $\tilde{\phi}X$ span a *$\tilde{\phi}$-section*. If the sectional curvatures $\tilde{K}(X)$ of all $\tilde{\phi}$-sections are independent of X, we say $\tilde{M}$ is of *constant $\tilde{\phi}$-sectional curvature*. It has been shown that in a normal contact manifold or a cosymplectic manifold of constant $\tilde{\phi}$-sectional curvature $\tilde{C}$,

$$\begin{aligned} \tilde{g}(\tilde{R}_{XY}Z, W) &= \alpha\{\tilde{g}(X, Z)\tilde{g}(Y, W) - \tilde{g}(X, W)\tilde{g}(Y, Z)\} \\ &+ \beta\{\tilde{\eta}(X)\tilde{\eta}(W)\tilde{g}(Z, Y) + \tilde{\eta}(Z)\tilde{\eta}(Y)\tilde{g}(X, W) - \tilde{\eta}(X)\tilde{\eta}(Z)\tilde{g}(Y, W) \\ &- \tilde{\eta}(Y)\tilde{\eta}(W)\tilde{g}(X, Z) + \tilde{\Phi}(X, W)\tilde{\Phi}(Z, Y) - \tilde{\Phi}(X, Z)\tilde{\Phi}(W, Y) \\ &+ 2\tilde{\Phi}(X, Y)\tilde{\Phi}(Z, W)\} \, , \end{aligned} \tag{2.3}$$

where $\alpha = (\tilde{C} + 3)/4$ and $\beta = (\tilde{C} - 1)/4$ is the normal contact case and $a = \beta = \tilde{C}/4$ in the cosymplectic case. This formula was shown for the normal contact case by Ogiue [5] and for the cosymplectic case by D. E. Blair (unpublished). We also note that the Ricci tensor is given by

$$\tilde{S}(X, Y) = \alpha^* \tilde{g}(X, Y) - \beta^* \tilde{\eta}(X)\tilde{\eta}(Y) \, , \tag{2.4}$$

where $\alpha^* = (n\alpha + \beta)2$ and $\beta^* = 2(n + 1)\beta$ in the normal contact case and $\alpha^* = \beta^* = 2(n + 1)\alpha$ in the cosymplectice case. Here the dimension of $\tilde{M}$ is assumed to be $2n + 1$.

3. Invariant submanifolds

Let M be a submanifold of codimension 2 imbedded in $\tilde{M}$ by $i\colon M \to \tilde{M}$. We will assume that M is invariant under $\tilde{\phi}$, i.e., for every tangent vector X of M there is a vector Y tangent to M such that $\tilde{\phi} i_* X = i_* Y$. Henceforth, we will use $X, Y, \cdots$ to represent tangent vectors to either M or $\tilde{M}$, the meaning being clear. Thus, there is a vector ξ tangent to M such that $i_* \xi = \tilde{\xi}$ (restricted to $i(M)$). It is easy to show that there are tensors ϕ, η and g defined on M by $\tilde{\phi} i_* X = i_* \phi X$, $\tilde{\eta}(i_* X) = \eta(X)$ and $\tilde{g}(i_* X, i_* Y) = g(X, Y)$. Then

$$i_*(\phi^2 X) = \tilde{\phi} i_* X = \tilde{\phi}^2 i_* X = -i_* X + \tilde{\eta}(i_* X)\tilde{\xi} = i_*(-X + \eta(X)\xi) .$$

Also, $\eta(\xi) = \tilde{\eta}(i_*\xi) = \tilde{\eta}(\tilde{\xi}) = 1$, $i_*(\phi\xi) = \tilde{\phi} i_* \xi = \tilde{\phi}\tilde{\xi} = 0$, and $\eta(\phi X) = \tilde{\eta}(i_* \phi X) = \tilde{\eta}(\tilde{\phi} i_* X) = 0$. We can then see that $g(\phi X, Y) = -g(X, \phi Y)$ and $g(\xi, \xi) = 1$. Thus, we have the following theorem.

Theorem 3.1 (*Yano & Ishihara* [7]). *(ϕ, ξ, η) is an almost contact structure on M with g as an associated metric.*

If we let $\Phi(X, Y) = g(X, \phi Y)$, then $\tilde{\Phi}(i_* X, i_* Y) = \tilde{g}(i_* X, \tilde{\phi} i_* Y) = \tilde{g}(i_* X, i_* \phi Y) = g(X, \phi Y) = \Phi(X, Y)$. From the coboundary formula we see that $d\eta(X, Y) = d\tilde{\eta}(i_* X, i_* Y)$ and also that $d\Phi(X, Y, Z) = d\tilde{\Phi}(i_* X, i_* Y, i_* Z)$. From these identities we see that $d\tilde{\eta} = \tilde{\Phi}$ implies that $d\eta = \Phi$. It is also straightforward to show that $[\tilde{\phi}, \tilde{\phi}](i_* X, i_* Y) = i_*[\phi, \phi](X, Y)$. Thus the following propositions are clear.

Proposition 3.2 (*Yano & Ishihara* [7]). *If $\tilde{\phi}$ is a normal contact structure on $\tilde{M}$, then ϕ is a normal contact structure on M.*

Proposition 3.3. *If $\tilde{\phi}$ is a cosymplectic structure on $\tilde{M}$, then ϕ is a cosymplectic structure on M.*

Let C be a unit vector field defined on $i(M)$ such that $\tilde{g}(C, i_* X) = 0$ and $\tilde{g}(\tilde{\phi} C, i_* X) = 0$ for all X. Since M is invariant, it follows that such a C can be found. Then we have

$$\tilde{\nabla}_{i_* X}(i_* Y) = i_*(\nabla_X Y) + H(X, Y)C + K(X, Y)\tilde{\phi} C , \tag{3.4}$$

where ∇ is the covariant derivative with respect to g, and H and K are symmetric tensors of type $(0, 2)$ on M. H and K are called the *second fundamental tensors* of M. Furthermore, we may write

$$\begin{aligned} \tilde{\nabla}_{i_* X} C &= -i_*(hX) + s(X)\tilde{\phi} C , \\ \tilde{\nabla}_{i_* X}(\tilde{\phi} C) &= -i_*(kX) - s(X) C , \end{aligned} \tag{3.5}$$

where s is a 1-form on M, $g(hX, Y) = H(X, Y)$, and $g(kX, Y) = K(X, Y)$.

Lemma 3.6. *The following identities hold*:

i) $H(X, Y) = K(X, \phi Y)$,

ii) $K(X, Y) = -H(X, \phi Y)$.

Proof.

$$\begin{aligned} (\tilde{\nabla}_{i_* X}\tilde{\phi}) i_* Y &= \tilde{\nabla}_{i_* X}(\tilde{\phi} i_* Y) - \tilde{\phi}(\tilde{\nabla}_{i_* X} i_* Y) \\ &= \tilde{\nabla}_{i_* X}(i_* \phi Y) - \tilde{\phi}(i_* \nabla_X Y + H(X, Y)C + K(X, Y)\tilde{\phi} C) \\ &= i_*(\nabla_X \phi Y) + H(X, \phi Y)C + K(X, \phi Y)\tilde{\phi} C - i_*(\phi \nabla_X Y) \\ &\quad - H(X, Y)\tilde{\phi} C - K(X, Y)(-C) . \end{aligned}$$

It can now be seen from (2.1) and (2.2) that $(\tilde{\nabla}_{i_*X}\tilde{\phi})i_*Y = i_*Z$ for some Z. The lemma then follows by noting that we have used the fact that $\tilde{\eta}(C) = 0$.

The identities of Lemma 3.6 show that

$$k = \phi h \,, \tag{3.7}$$

$$h\phi = -\phi h \,. \tag{3.8}$$

From this it follows that tr h = tr $k = 0$. Here tr h denotes the trace of h. We also note that $H(X, \xi) = 0$ and $K(X, \xi) = 0$ for all X.

The following lemma is proved in [6].

Lemma 3.9. *Let V be a $2n$-dimensional real vector space with a complex structure J and a positive definite inner product g which is hermitian (i.e., $J^2 = -I$ and $g(JX, JY) = g(X, Y)$). If A is symmetric with respect to g and $AJ = -JA$, there exists an orthonormal basis $\{e_1, \cdots, e_n, Je_1, \cdots, Je_n\}$ of V with respect to which the matrix of A is diagonal of the form*

$$\begin{bmatrix} \lambda_1 & & & & & \\ & \ddots & & & & \\ & & \lambda_n & & & \\ & & & -\lambda_1 & & \\ & & & & \ddots & \\ & & & & & -\lambda_n \end{bmatrix}$$

This lemma and equation (3.8) then show that at each point m of M there is an orthonormal basis $\{\xi, e_1, \cdots, e_{n-1}, \phi e_1, \cdots, \phi e_{n-1}\}$ of M_m, the tangent space of M at m, such that h at m is diagonal of the form

$$\begin{bmatrix} \lambda_1 & & & & & & \\ & \ddots & & & & & \\ & & \lambda_{n-1} & & & & \\ & & & -\lambda_1 & & & \\ & & & & \ddots & & \\ & & & & & -\lambda_{n-1} & \\ & & & & & & 0 \end{bmatrix} \tag{3.10}$$

with respect to this basis.

4. Main Theorems

The following Gauss-Codazzi equation for the curvature operator of M is well-known and follows directly from equations (3.4) and (3.5).

$$(4.1)\quad \begin{aligned}\tilde{R}_{i_*Xi_*Y}i_*Z = i_*[R_{XY}Z &- (H(Y,Z)hX - H(X,Z)hY) \\ &- (K(Y,Z)kX - K(X,Z)kY)] + g((\nabla_X h)Y - (\nabla_Y h)X \\ &- s(X)kY + s(Y)kX, Z)C + g((\nabla_X k)Y - (\nabla_Y k)X \\ &+ s(X)hY - s(Y)hX, Z)\tilde{\phi}C .\end{aligned}$$

From this it follows that

$$(4.2)\quad \begin{aligned} S(X, Y) = \tilde{S}(i_*X, i_*Y) &+ \operatorname{tr} h\, H(X, Y) - g(hX, hY) \\ &+ \operatorname{tr} k\, K(X, Y) - g(kX, kY) ,\end{aligned}$$

where S is the Ricci tensor on M. Because of Lemma 3.6, equation (4.2) simplifies to

$$(4.2)'\quad S(X, Y) = \tilde{S}(i_*X, i_*Y) - 2g(h^2X, Y) .$$

Lemma 4.3. *If $\tilde{M}$ is a cosymplectic manifold of constant $\tilde{\phi}$-sectional curvature, then $\nabla_X h^2 = 0$ implies that $\nabla_X S = 0$.*

Proof. Using equation (2.4), equation (4.2)′ simplifies

$$S(X, Y) = \frac{(n + 1)\tilde{C}}{2}(g(X, Y) - \eta(X)\eta(Y)) - 2g(h^2X, Y) ,$$

from which the lemma follows.

If we assume $\tilde{M}$ is of constant $\tilde{\phi}$-sectional curvature, then (2.3) can be used to show that $\tilde{R}_{i_*Xi_*Y}i_*Z$ is in fact tangent to M. Hence, the coefficients of C in (4.1) must vanish, i.e.,

$$(4.3)\quad (\nabla_X h)Y - (\nabla_Y h)X - s(X)kY + s(Y)kX = 0 .$$

The vanishing of the coefficient of $\tilde{\phi}C$ adds nothing new. M is said to be *totally geodesic* if $H = K = 0$.

Theorem 4.4. *M is totally geodesic if and only if M is of constant ϕ-sectional curvature.*

Proof. Let X be a vector orthogonal to ξ. Then from (4.1), we have that

$$\begin{aligned} g(R_{X\phi X}X, \phi X) &= \tilde{g}(\tilde{R}_{i_*X\tilde{\phi}i_*X}\tilde{\phi}i_*X, i_*X) + H(\phi X, X)H(X, \phi X) \\ &\quad + H(X, X)H(\phi X, \phi X) + K(\phi X, X)K(X, \phi X) \\ &\quad + K(X, X)K(\phi X, \phi X) \\ &= \tilde{g}(\tilde{R}_{i_*X\tilde{\phi}i_*X}\tilde{\phi}i_*X, i_*X) + 2(H^2(X, X) + K^2(X, X)) .\end{aligned}$$

Now $\tilde{g}(i_*X, \tilde{\xi}) = g(X, \xi)$ so that if X is orthogonal to ξ then i_*X is orthogonal to $\tilde{\xi}$. Hence, $H = K = 0$ implies that M is of constant ϕ-sectional curvature $\tilde{c}$.

Now assume that M is of constant ϕ-sectional curvature. Then $S(X, Y) =$

$\bar{\alpha}^* g(X, Y) - \bar{\beta}^* \eta(X)\eta(Y)$ for constants $\bar{\alpha}^*$ and $\bar{\beta}^*$ by (2.4). Thus, by (4.2)′,

$$h^2 = aI + b\xi \otimes \eta \tag{4.5}$$

for appropriate constants a and b. Since $h\xi = 0$, we see that $a + b = 0$. Let $X = (e_i + \phi e_j)/\sqrt{2}$, where $i \neq j$ and the e_i's are from the basis for M_m mentioned after Lemma 3.9. Then $g(X, X) = 1$ and it can be shown that $g(R_{X\phi X}X, \phi X) = \tilde{c}$. This shows that $H(X, X) = 0$ and $K(X, X) = 0$ for all X. However, since H and K are symmetric, we have that $H = K = 0$ and the proof is finished.

Definition 4.6. Let (ϕ, ξ, η, g) be an almost contact metric structure on a manifold M. Then M is said to be *η-Einsteinian* if $S = ag + b\eta \otimes \eta$ for some a and b, necessarily constants, where S is the Ricci tensor of M.

Definition 4.7. A manifold M is *locally symmetric* if $\nabla_X R = 0$ for all X.

Proposition 4.8. *If M is a normal contact η-Einsteinian but not Einsteinian manifold, then M is not locally symmetric.*

Proof. Certainly if $\nabla_X R = 0$ then $\nabla_X S = 0$. However, from Definition 4.6,

$$(\nabla_X S)(Y, Z) = b(\nabla_X \eta)(Y)\eta(Z) + b\eta(Y)(\nabla_X \eta)(Z) .$$

Therefore, since $(\nabla_X \eta)(Y) = d\eta(X, Y)$ and $d\eta(\xi, X) = 0$ for all X, we have that

$$(\nabla_X S)(Y, \xi) = bd\eta(X, Y) \neq 0 .$$

Note that if M is of constant ϕ-sectional curvature 1, then M is in fact of constant curvature. Thus, we have the following crollary.

Corollary 4.9. *If M is a normal contact manifold of constant ϕ-sectional curvature $\neq 1$, then M is not locally symmetric.*

We now proceed to prove our main theorem.

Theorem 4.10. *If $\tilde{M}$ is a cosymplectic manifold of constant $\tilde{\phi}$-sectional curvature and M is an invariant submanifold of codimension 2 of $\tilde{M}$ which is η-Einsteinian, then M is locally symmetric.*

Lemma 4.11.

$$\nabla_X h = s(X)k .$$

Proof of Lemma 4.11. By (4.3) we have that

$$(\nabla_\xi h)Y - (\nabla_Y h)\xi - s(\xi)kY = 0 .$$

However, $(\nabla_Y h)\xi = \nabla_Y(h\xi) - h\nabla_Y \xi = 0$. Thus $\nabla_\xi h = s(\xi)k$. If X is orthogonal to ξ, the proof of Proposition 7 of [6] and the fact that $(\nabla_X h)\xi = 0$ show that $\nabla_X h = s(X)k$.

Now, since $k = \phi h$, we see that

$$\nabla_X k = \nabla_X(\phi h) = \phi \nabla_X h = s(X)\phi k = s(X)\phi^2 h = -s(X)h .$$

The following lemma is proved in [6].

Lemma 4.12. *If M is an arbitrary Riemannian manifold with metric g, then the tensor field P defined on M by*

$$P(X, Y, Z, W) = g(BX, Z)g(BY, W) ,$$

where B is a tensor field of type $(1, 1)$ *on M, has covariant derivative given by*

$$(\nabla_V P)(X, Y, Z, W) = g((\nabla_V B)X, Z)g(BY, W) + g(BX, Z)g((\nabla_V B)Y, W) .$$

Proof of Theorem 4.11. Now let $\tilde{R}(X, Y, Z, W) = \tilde{g}(\tilde{R}_{XY}Z, W)$. By equation (2.3), we see that $(\tilde{\nabla}_V \tilde{R})(X, Y, Z, W) = 0$ since $\tilde{\nabla}_V \tilde{\Phi} = 0$ and $\tilde{\nabla}_X \tilde{\eta} = 0$. Let

$$\begin{aligned} D(X, Y, Z, W) = {} & g(hX, W)g(hY, Z) - g(hY, W)g(hX, Z) \\ & + g(kX, W)g(kY, Z) - g(kY, W)g(kX, Z) , \end{aligned}$$

so that $\tilde{R}(i_*X, i_*Y, i_*Z, i_*W) = i_*(R(X, Y, Z, W) - D(X, Y, Z, W))$. Hence, by Lemma 4.12,

$$\begin{aligned} (\nabla_V D)(X, Y, Z, W) &= g((\nabla_V h)X, W)g(hY, Z) + g(hX, W)g((\nabla_V h)Y, Z) \\ &\quad - g((\nabla_V h)Y, W)g(hX, Z) - g(hY, W)g((\nabla_V h)X, Z) \\ &\quad + g((\nabla_V k)X, W)g(kY, Z) + g(kX, W)g((\nabla_V k)Y, Z) \\ &\quad - g((\nabla_V k)Y, W)g(kX, Z) - g(kY, W)g((\nabla_V k)X, Z) \\ &= s(V)\{g(kX, W)g(hY, Z) + g(hX, W)g(kY, Z) \\ &\quad - g(kY, W)g(hX, Z) - g(hY, W)g(kX, Z) \\ &\quad - g(hX, W)g(kY, Z) - g(kX, W)g(hY, Z) \\ &\quad + g(hY, W)g(kX, Z) + g(kY, W)g(hX, Z)\} \\ &= 0 . \end{aligned}$$

Thus, the proof is finished.

Assume now that $\tilde{M}$ is a normal contact manifold. Again we have that $\tilde{R}(i_*X, i_*Y, i_*Z, i_*W) = i_*(R(X, Y, Z, W) - D(X, Y, Z, W))$. If $\tilde{M}$ is of constant curvature, then $\tilde{\nabla}_V \tilde{R} = 0$. (If we merely assume that $\tilde{M}$ is of constant $\tilde{\phi}$-sectional curvature then $\tilde{\nabla}_V \tilde{R}$ can be computed. It turns out to be a rather long expression involving the $\tilde{\Phi}$, $\tilde{\eta}$ and $\tilde{g}$. Since we are interested in $(\tilde{\nabla}_{i_*V} \tilde{R})(i_*X, i_*Y, i_*Z, i_*W)$, this can be expressed in terms of $\tilde{\Phi}$, η and g.) If M is Einsteinian, then $(4.2)'$ shows that $g(h^2X, Y) = \lambda g(X, Y)$ for some λ. However, since $h\xi = 0$, we have $h^2 = 0$ and hence $h = 0$. Also $k = 0$ so that M is totally geodesic and hence $D = 0$. Thus, $\nabla_V R = 0$ (see [7]). It is slightly more complicated to consider the case where M is η-Einsteinian. In this case we have that $\nabla_V R \neq 0$ (see [7]).

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Michigan State University